



Numerical-experimental assessment of Stress Intensity Factors in Ultrasonic Very-High-Cycle Fatigue (VHCF)

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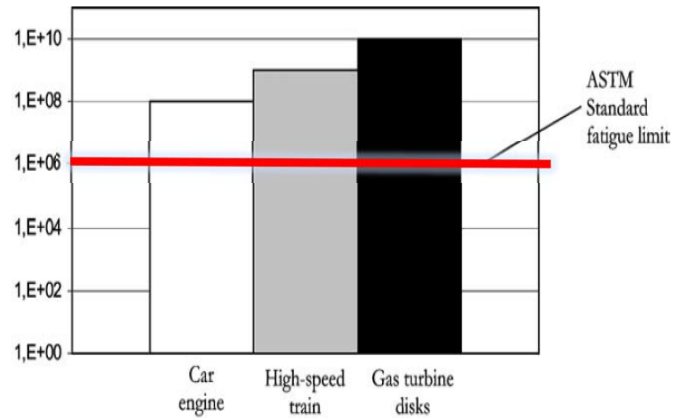
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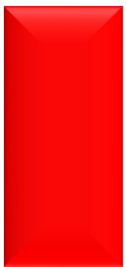


VHCF: Introduction

► New needs:



232 Days



50 Hz

116 Days

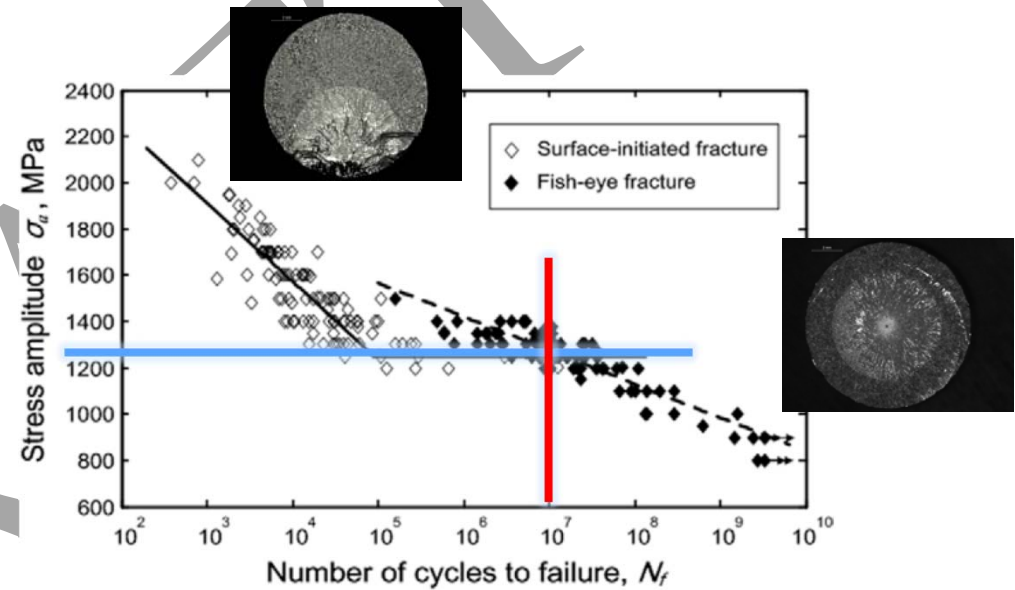


100 Hz

14 Hour



20 kHz



► New trends: no fatigue limit?

► New tests: ultrasonic VHCF



Objects

- Fracture toughness is a very important (critical) material parameter in VHCF.
- Nowadays many researchers evaluate the stress intensity factor in VHCF regime by using literature analytical equations validated in the static field. The validity of these equations in the dynamic field is an open question.
- Objective of present research is to search for a robust method to easily and effectively evaluate the Stress Intensity Factor(SIF) in the dynamic field (VHCF tests).
- Creating a strong and powerful *tool(package)* to calculate Stress Intensity Factor by different methods in Very High cycle Fatigue regime.



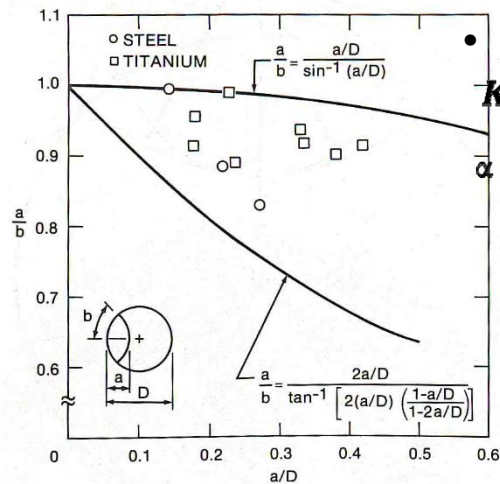
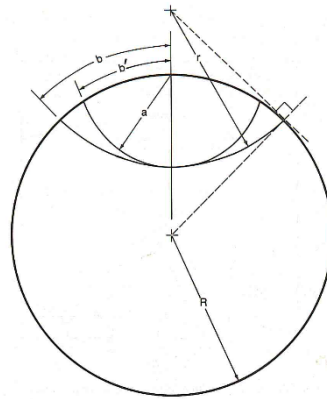
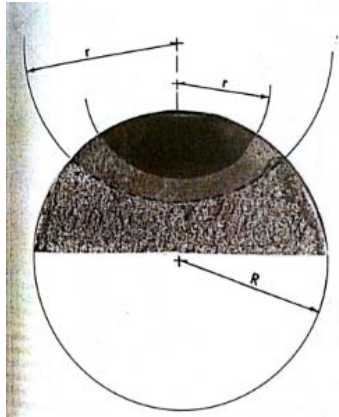
Outline

- *Analytic solutions: state of the art.*
- *FEM solutions: state of the art.*
- *Material & Experimental methods.*
- *FEM predictions in VHCF.*
- *Computational cost: Crisis.*
- *Hybrid Method: The present study.*
- *Solver: Harmonic Balance Method.*
- *Reduced order model (ROM) : Dynamic Sub-structuring (CB-CMS).*
- *SIF evaluation :Virtual Crack Closure Technique(VCCT).*
- *SIF evaluation :J-Integral.*
- *SIF evaluation :Equivalent Domain Integral.*
- *SIF evaluation :Interaction Integral (M-Integral).*
- *Results for a simple bar as a bench mark.*
- *Application of Hybrid Method on Hourglass and Gaussian Samples*
- *Conclusions .*



Analytic solutions: state of the art

Forman solution

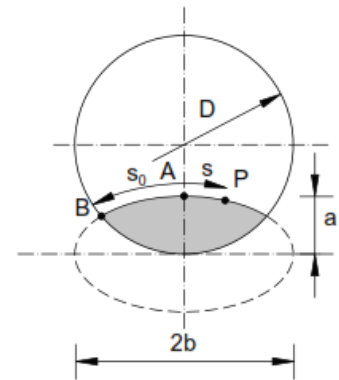
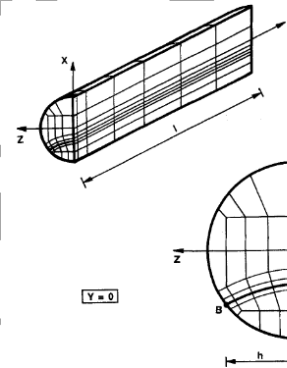


One-parameter K -solution

$$K_I = \sigma_T F_T(\alpha) \sqrt{\pi a}$$

$$\alpha = \frac{a}{2R} \text{ crack depth ratio}$$

Shih & Cai solution



Two-parameter K -solution

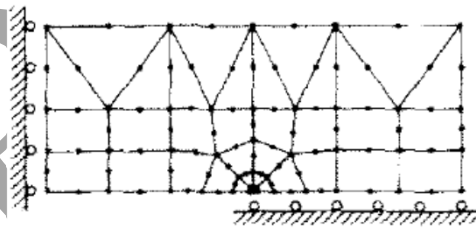
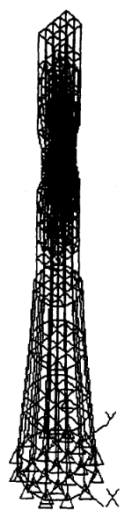
$$K_{ijk} = \sigma \sqrt{\pi a} Y_{ijk} \left(\frac{a}{b}, \alpha \right)$$

$$Y_{LF} \left(\frac{a}{b}, \alpha \right) = 0.582 - 0.115 \frac{a}{b} + 5.39\alpha - 0.124 \left(\frac{a}{b} \right)^2 \\ - 1.65 \frac{a}{b} \alpha - 22.01\alpha^2 + 1.65 \left(\frac{a}{b} \right)^2 \alpha \\ + 7.58 \frac{a}{b} \alpha^2 + 49.40\alpha^3 - 7.42 \left(\frac{a}{b} \right)^2 \alpha^2 \\ - 6.44 \frac{a}{b} \alpha^3 - 22.78\alpha^4$$

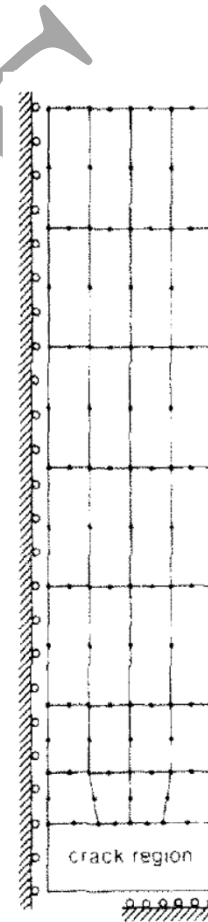


FEM solutions: state of the art

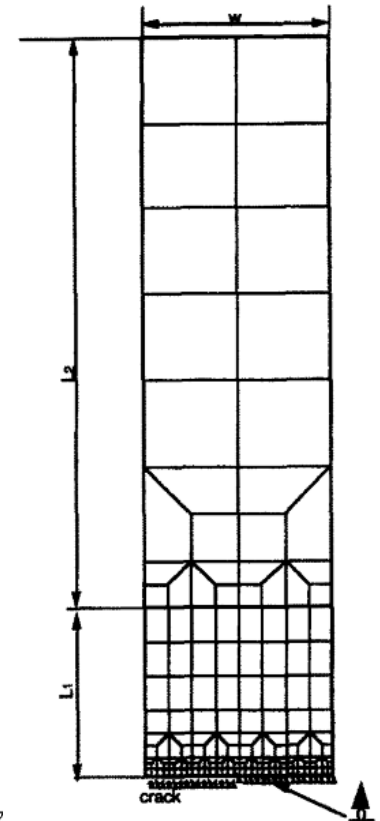
- Most of them in **2D geometries**
- **Poisson's ratio neglected**
- Crack in mid-plane (perfectly **symmetric case**)
- **Contact** between crack surfaces **neglected**



crack region



crack region





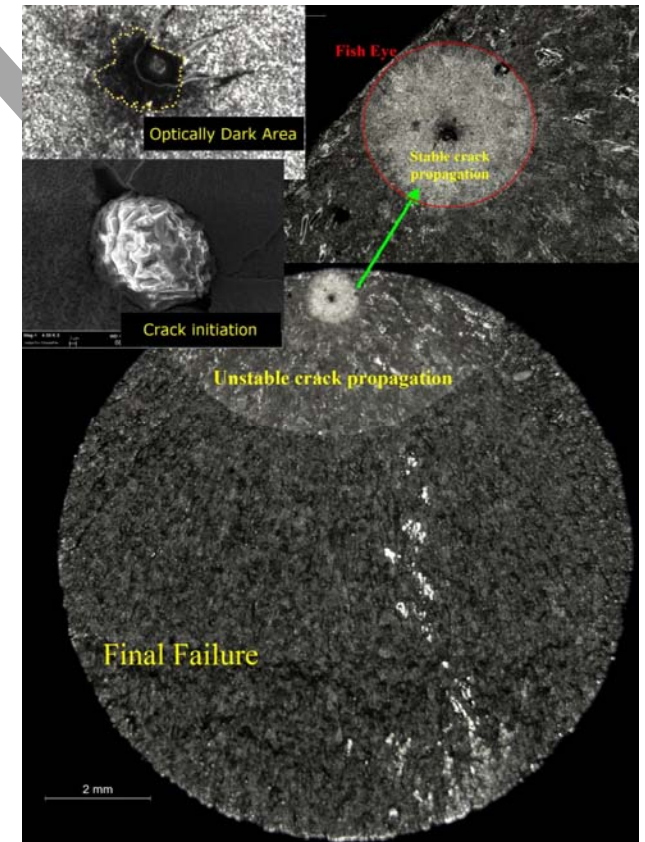
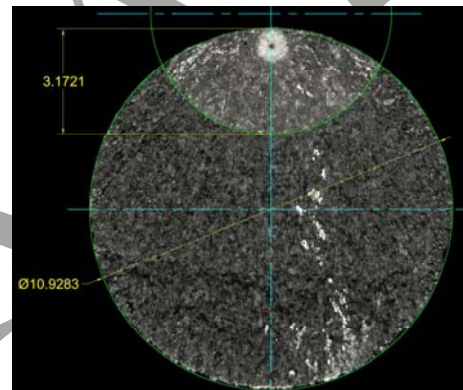
Material & Experimental methods: present study

- **H13 steel** quenched and tempered
- Two specimen geometries: **Hourglass & Gaussian**
- Constant stress amplitude with $R=-1$ at 20 kHz
- Fractography through optical microscope
- Measurement with CAD software

Typical analysis %	C	Si	Mn	Cr	Mo	V
	0.39	1.0	0.4	5.3	1.3	0.9
Standard specification	AISI H13, W.-Nr. 1.2344, EN X40CrMoV5-1					

These dimensions were used for FEM simulations

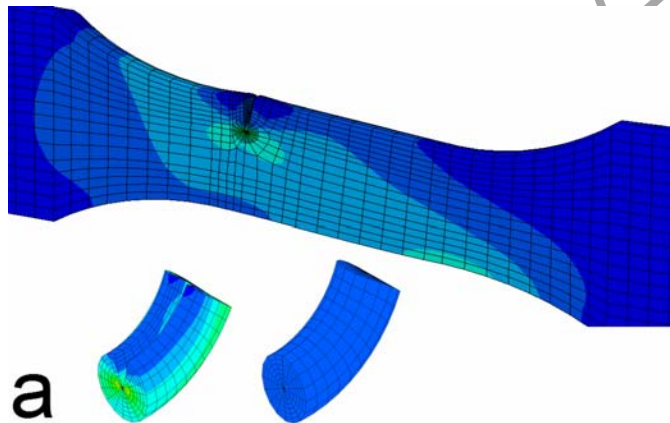
Sample name	Diameter r [μm]	Depth h [μm]	Arc length [μm]	Crack radius [μm]	Displacement amplitude [μm]	σ_a [MPa]
Hourglass	5995	2524	2817	5820	44	640
Gaussian	10928	3172	3516	3624	52	510



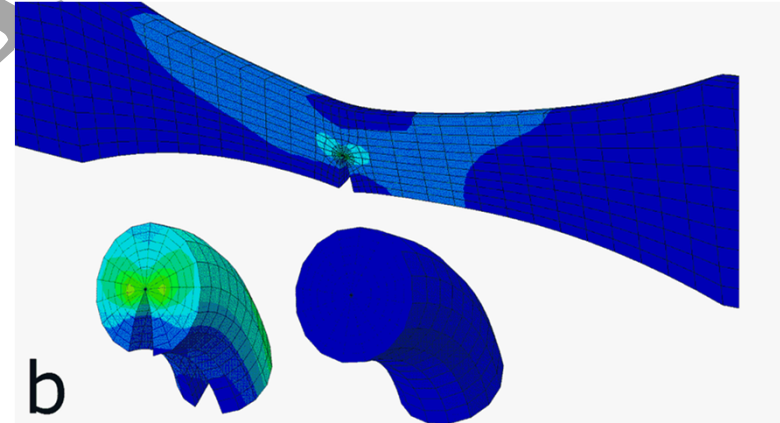


FEM predictions in VHCF

- Commercial software **ABAQUS: 3D model with implicit solver** (time step $10^{-6}s$)
- Two geometries: **Hourglass** (4828 elements) and **Gaussian** (6192 elements)
- **Seam configuration** for crack modeling
- Crack tip: **collapsed Barsoum elements** (16 elements in hoop direction and 5 contours)
- Penalty method to model the **contact between crack surfaces**
- **M-integral** for SIF computation



a): Gaussian

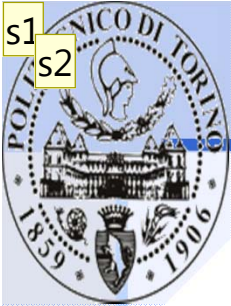


b): Hourglass



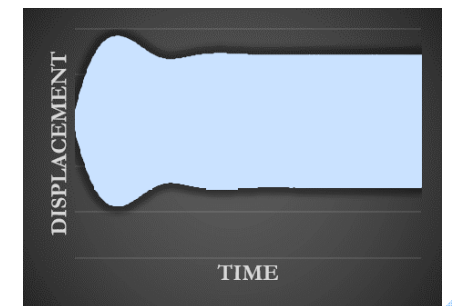
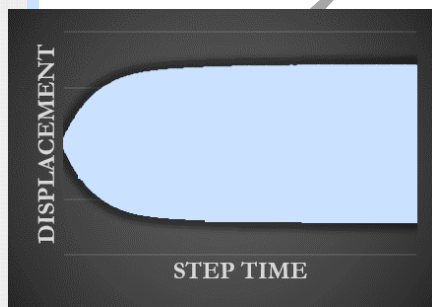
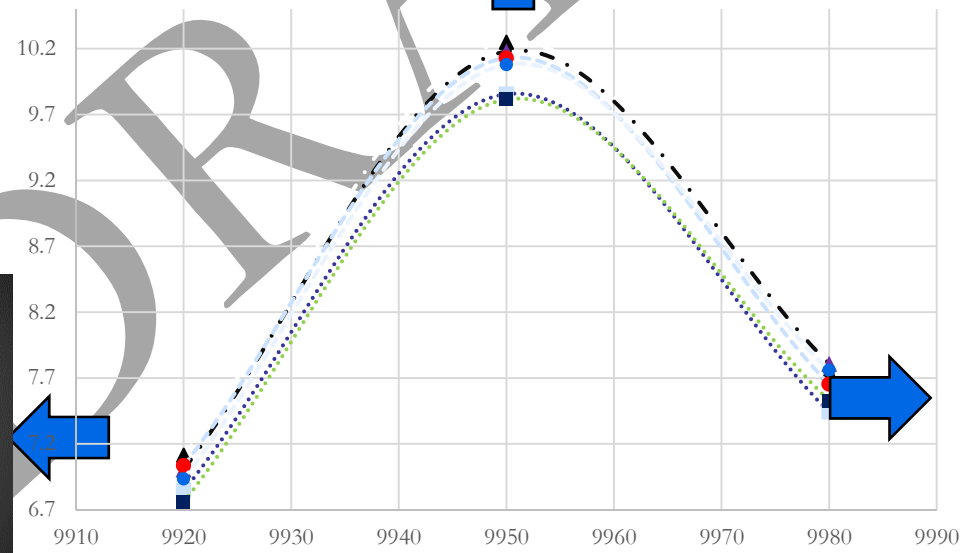
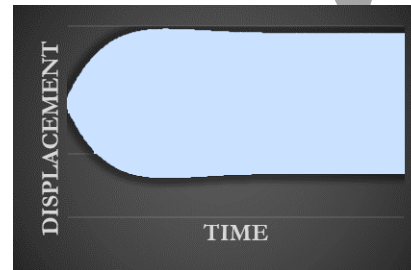
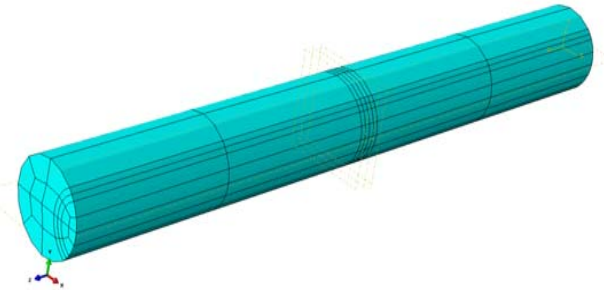
Computational cost: Crisis!!

- Direct time integration method (Implicit) used in this study as reference solution (Time-domain analysis (TDA)).
- The direct integration method used by ABAQUS is the Newmark Alpha method (Hilber-Hughes-Taylor scheme).
- The Newmark Alpha method is unconditionally stable.
- In order to obtain the fine high-frequency detail of the response it is desirable to use the fine step time. ($10^{-6} \sim 10^{-7}$).
- The solution is in steady state regime.
- Solution time for 3D specimen is drastically long, computational cost is so high and this solution method is not efficient!!!!



Computational cost: Crisis!!

- 10KHz frequency
- Surfaced cracked bar with 192 element





Hybrid Method: The present study

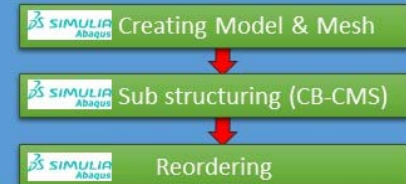
Suggestions to reduce the computational cost for this study:

- More efficient solver to solve the nonlinear problem fast, reliable and accurate :
 - ❖ Harmonic Balance Method (HBM)
 - ☐ Reduces solution time.
- Reduce the nonlinear core size of the problem by Dynamic Sub-structuring:
 - ❖ Component Mode Synthesis- Craig-Bampton Method (CB-CMS)
 - ☐ Drastically reduced the size of mass, stiffness and damping matrices.
- Simplest method to evaluate Stress Intensity Factor :
 - ❖ Virtual Crack Closure Techniques (VCCT), J-Integral, and *M-Integral*
 - ☐ Ease of use and implement.

Package Flow-chart



PRE-PROCESSING

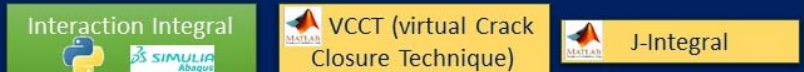


ANALYSIS



Post-Processing

➤ Stress Intensity Factor



Visualization





Dynamic Sub-structuring: Craig-Bampton

- In numerous cases of nonlinear systems having a large numbers of DOFs, the actual nonlinear components are spatially localized (like a crack).

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F_E\} + \{F_{NL}(X, \dot{X})\}$$

$$\{X\} = \begin{Bmatrix} X_L \\ X_N \end{Bmatrix} \quad \{F_{NL}\} = \begin{Bmatrix} 0 \\ F_N(X_N, \dot{X}_N) \end{Bmatrix} \quad \{F_E\} = \begin{Bmatrix} F_{E,L} \\ F_{E,N} \end{Bmatrix}$$

F_E : Excitation force.

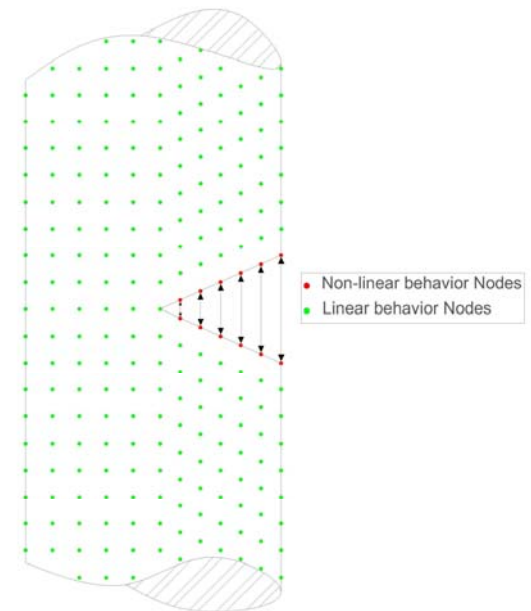
F_{NL} : Non-linear forces.

X_L : linear DOFs

X_N : Non – linear DOFs

$[M], [K], [C]$: mass , stiffness and damping Matrices.

$$\begin{bmatrix} M_{LL} & M_{LN} \\ M_{NL} & M_{NN} \end{bmatrix} \begin{Bmatrix} \ddot{X}_L \\ \ddot{X}_N \end{Bmatrix} + \begin{bmatrix} C_{LL} & C_{LN} \\ C_{NL} & C_{NN} \end{bmatrix} \begin{Bmatrix} \dot{X}_L \\ \dot{X}_N \end{Bmatrix} + \begin{bmatrix} K_{LL} & K_{LN} \\ K_{NL} & K_{NN} \end{bmatrix} \begin{Bmatrix} X_L \\ X_N \end{Bmatrix} = \begin{Bmatrix} F_{E,L} \\ F_{E,N} \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_N(X_N, \dot{X}_N) \end{Bmatrix}$$





Dynamic Sub-structuring: Craig-Bampton

In the FEM of such cases, CB-CMS can be used to reduce the size of the system. Then, only the nonlinear nodes are retained in the nonlinear governing equations.

$$\{X\} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} \begin{pmatrix} \psi \\ I \end{pmatrix} \begin{Bmatrix} q_L \\ X_N \end{Bmatrix}$$

Normal modes
with fixed
interface

Constraint
modes

$$[T] = \begin{pmatrix} \varphi & \psi \\ 0 & I \end{pmatrix}$$

Craig-Bampton transformation matrix

$$\psi = -K_{LL}^{-1} K_{LN}$$

$$\begin{bmatrix} m_{LL} & m_{LN} \\ m_{NL} & m_{NN} \end{bmatrix} \begin{Bmatrix} \ddot{q}_L \\ \ddot{X}_N \end{Bmatrix} + \begin{bmatrix} c_{LL} & c_{LN} \\ c_{NL} & c_{NN} \end{bmatrix} \begin{Bmatrix} \dot{q}_L \\ \dot{X}_N \end{Bmatrix} + \begin{bmatrix} k_{LL} & k_{LN} \\ k_{NL} & k_{NN} \end{bmatrix} \begin{Bmatrix} q_L \\ X_N \end{Bmatrix} = \begin{Bmatrix} f_{E,L} \\ f_{E,N} \end{Bmatrix} + \begin{Bmatrix} 0 \\ f_N(X_N, \dot{X}_N) \end{Bmatrix}$$

$$[m]_{CB} = [T]^T [M] [T]$$

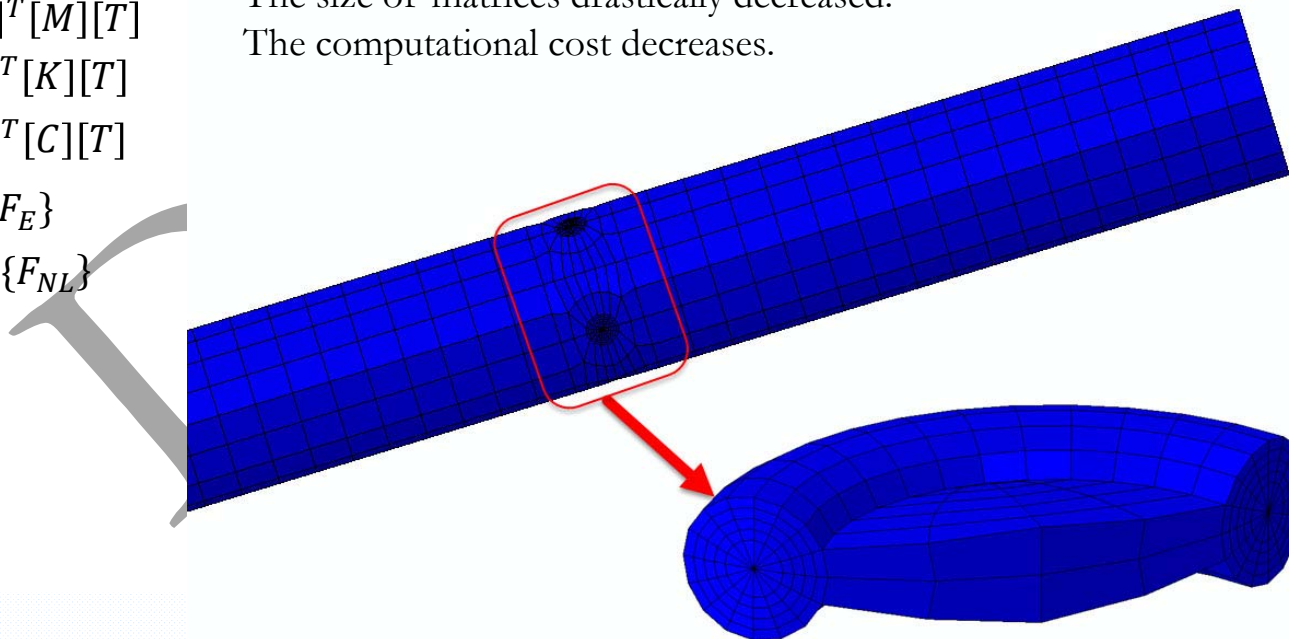
$$[k]_{CB} = [T]^T [K] [T]$$

$$[c]_{CB} = [T]^T [C] [T]$$

$$\{f_E\} = [T] \{F_E\}$$

$$\{f_{NL}\} = [T] \{F_{NL}\}$$

The size of matrices drastically decreased.
The computational cost decreases.





Nonlinear Solver: Harmonic Balance Method

Time Domain to Frequency Domain

- HBM is by far the most computationally efficient method for obtaining the steady-state solution to nonlinear dynamics problem.

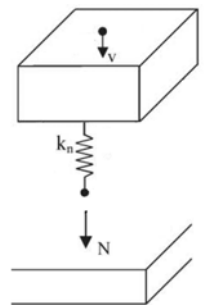
$$X(t) = \sum_{n=0}^{n=H} \bar{x}^{(n)} e^{in\omega t} \quad f_E(t) = \sum_{n=0}^{n=H} \bar{f}_E^{(n)} e^{in\omega t} \quad f_{NL}(t) = \sum_{n=0}^{n=H} \bar{f}_{NL}^{(n)} e^{in\omega t}$$

$$\left[-(n\omega)^2 \begin{pmatrix} m_{LL} & m_{LN} \\ m_{NL} & m_{NN} \end{pmatrix}^{(n)} + in\omega \begin{pmatrix} c_{LL} & c_{LN} \\ c_{NL} & c_{NN} \end{pmatrix}^{(n)} + \begin{pmatrix} k_{LL} & k_{LN} \\ k_{NL} & k_{NN} \end{pmatrix}^{(n)} \right] \begin{Bmatrix} \bar{q}_L^{(n)} \\ \bar{x}_N^{(n)} \end{Bmatrix} = \begin{Bmatrix} \bar{f}_{E,L}^{(n)} \\ \bar{f}_{E,N}^{(n)} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \bar{f}_{NL,N}^{(n)} \end{Bmatrix}$$

H : number of harmonics.

Harmonic Balance-Frequency Domain
(New nonlinear Equation)

- Solving of this new nonlinear equation is possible by iterative solution methods like Newton-Raphson.
- \mathbf{F}_{NL} : Non-linear forces in our case is only the normal contact force.
 - ✓ AFT method applied to calculate contact forces in time domain and send back to frequency domain.
 - ✓ Node-to-node frictionless contact element with normal contact stiffness (penalty).
 - ✓ Compression force $N = \max(k_n v, 0)$, with v : normal relative displacement.





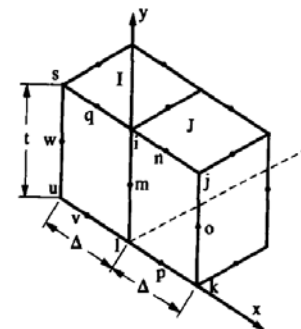
- $$G_I = \lim_{\Delta a \rightarrow 0} \frac{1}{2b \cdot \Delta a} \int_0^b \int_0^{\Delta a} w_R(r, y) \cdot \sigma_z(\Delta a - r, y) dr dy$$

$$\sigma_z = \frac{A_1}{\sqrt{x}} + A_2 + A_3 y + A_4 \sqrt{x} + A_5 \frac{y}{\sqrt{x}} + A_6 y^2 + A_7 \sqrt{xy} + A_8 \frac{y^2}{\sqrt{x}}$$

A diagram illustrating a crack in a mesh. The crack is shown as a dashed line, with the crack tip labeled "Crack Tip". The crack surfaces are labeled "Crack Surfaces". The mesh is divided into an "Intact Area" and a "Crack Area". The crack length is labeled Δa . The crack is oriented along the X-axis. The crack tip is at the origin of the X-Y coordinate system. The crack surfaces are labeled K_I , K_{II} , and K_{III} . The crack tip is labeled L_j , L_i , L_m , and L_l . The crack is labeled b .

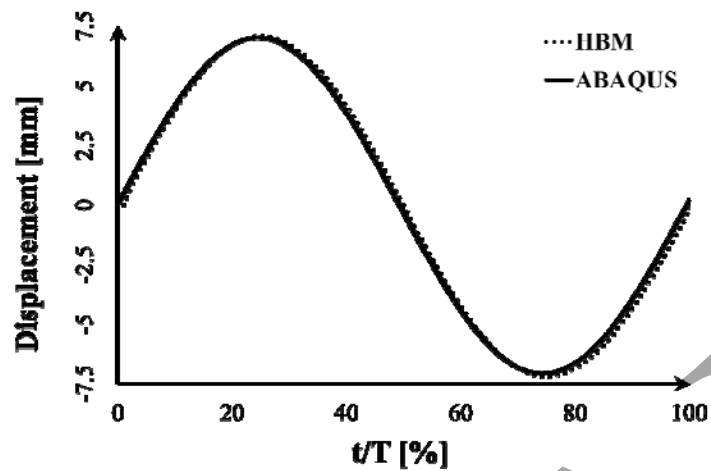
- $$G_I = -\frac{1}{2 \times \Delta a \times b} \left[\frac{1}{2} F_{Ki} (w_{Kl} - w_{Kl}^*) + F_{Li} (w_{Ll} - w_{Ll}^*) + F_{Lj} (w_{Lm} - w_{Lm}^*) + \frac{1}{2} F_{Mi} (w_{Ml} - w_{Ml}^*) \right]$$

- $$K_I = \sqrt{[G_I/(1 - v^2)]}$$

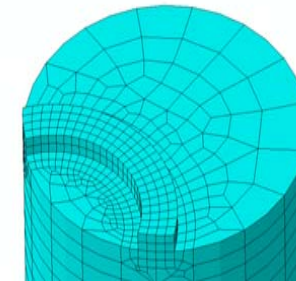
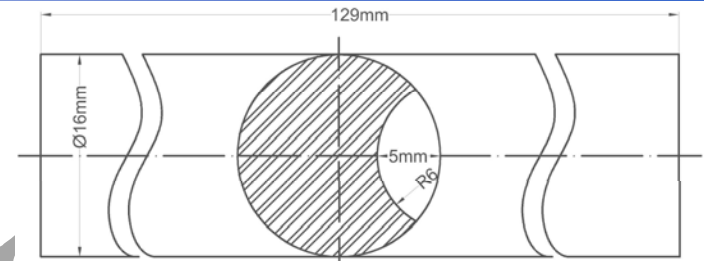




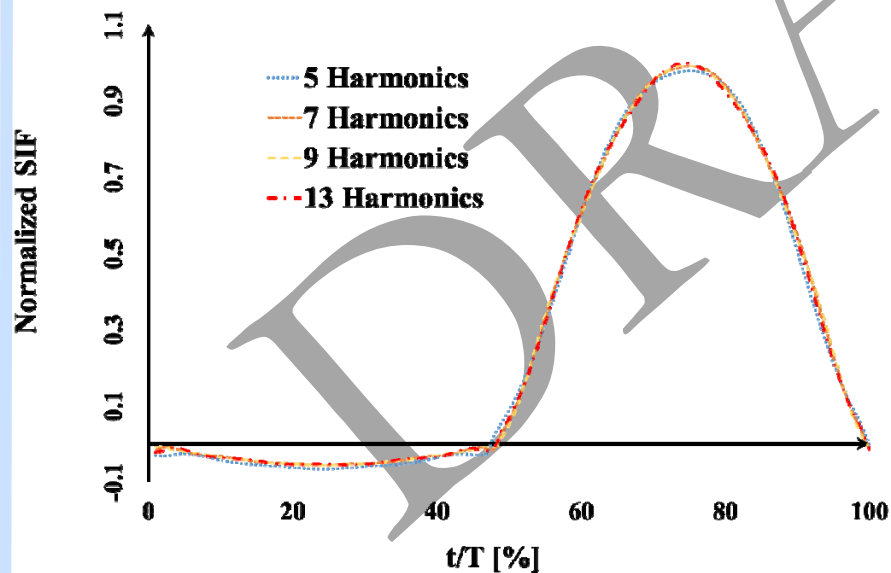
Results for a simple bar: VCCT



shows the displacement of the one node during a cycle



Geometry and mesh of the cracked bar



the computed SIFs during a cycle, according to different numbers of harmonics

	Implicit solver ABAQUS	HBM+VCCT	Reduction ratio
Computational time [s]	93600	158	~600

Comparison between computational costs



Stress Intensify Factor evaluation: Contour Integral(J-Integral)

- The energy release rate is described, J. Rice obtained a path independent contour integral, the J-integral, from his first-name Jim, which describe the energy release rate in LEFM, and the J-integral is equivalent to G:

$$J=G$$

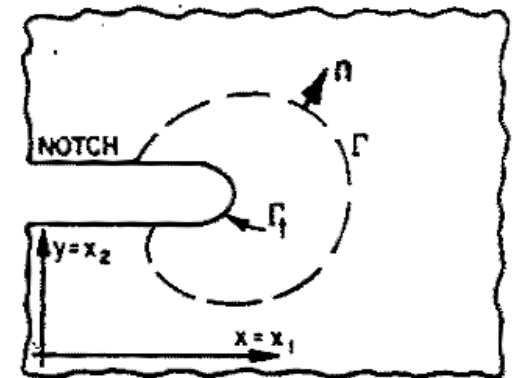
- The J-integral can be described by a path around a crack tip given by followed expression in 2D as path integral:

$$J = \oint_{\Gamma} (W dy - T_i \frac{\partial u_i}{\partial x} ds)$$

$$W = \int \frac{1}{2} \sigma_{ij} \epsilon_{ij}$$

$$T_i = \sigma_{ij} n_j$$

Component	Description
w	Strain energy density
T_i	Traction vector
n_j	Unit vector normal to Γ
u_i	Displacement vector
ds	Length increment along the contour Γ
Γ	Path around crack tip
$\sigma_{ij}, \epsilon_{ij}$	Stress and strain tensor



- The J-integral is effective for evaluating K in two-dimensional crack problems.

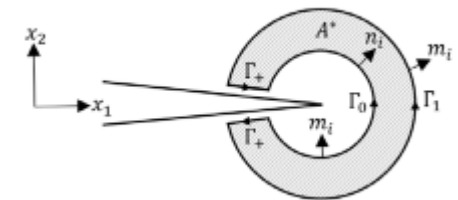
For three-dimensional problems, however, it is difficult to distinguish K at different x_3 locations, assuming the line integral is performed on the x_1 - x_2 plane. Thus an alternative procedure needs to be developed to determine the distribution of K through the thickness.



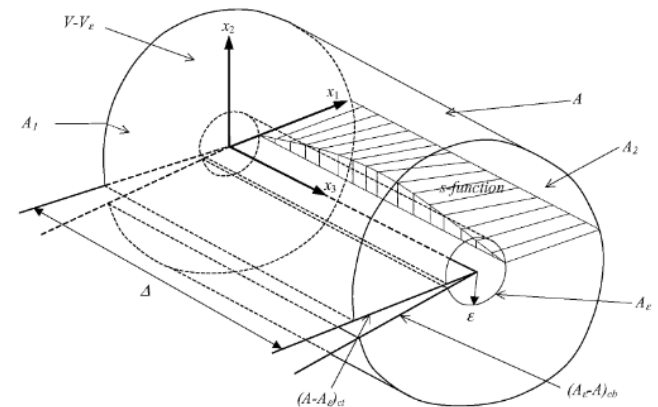
Stress Intensity Factor evaluation: Contour Integral(Equivalent Domain Integral)

- The Domain Integral is a numerical solution of the J-integral .
- A closed contour containing an inner Γ_0 and outer Γ_1 contours is used where the inner is vanishingly small and the outer is finite .
- Divergence theorem can be applied and instead of integrating around a path of the crack tip, integration over the area between the paths is used to evaluate the J-Integration.

$$J = \int_{A^*} \left[\delta_{ij} \frac{\partial u_j}{\partial x_1} - W \delta_{ij} \right] \frac{\partial q}{\partial x} dA - \int_{\Gamma} \delta_{2j} \frac{\partial u_j}{\partial x_1} q d\Gamma$$



Component	Description
w	Strain energy density
q	Weight function in A^* , by $\Gamma_0 \rightarrow q = 1$ and $\Gamma_1 \rightarrow q = 0$
u_j	Displacement vector components
δ_{ij}	Kronecker delta
Γ_0, Γ_1	Inner and outer contours
Γ_+, Γ_-	Upper and lower crack face
σ_{ij}	Stress tensor



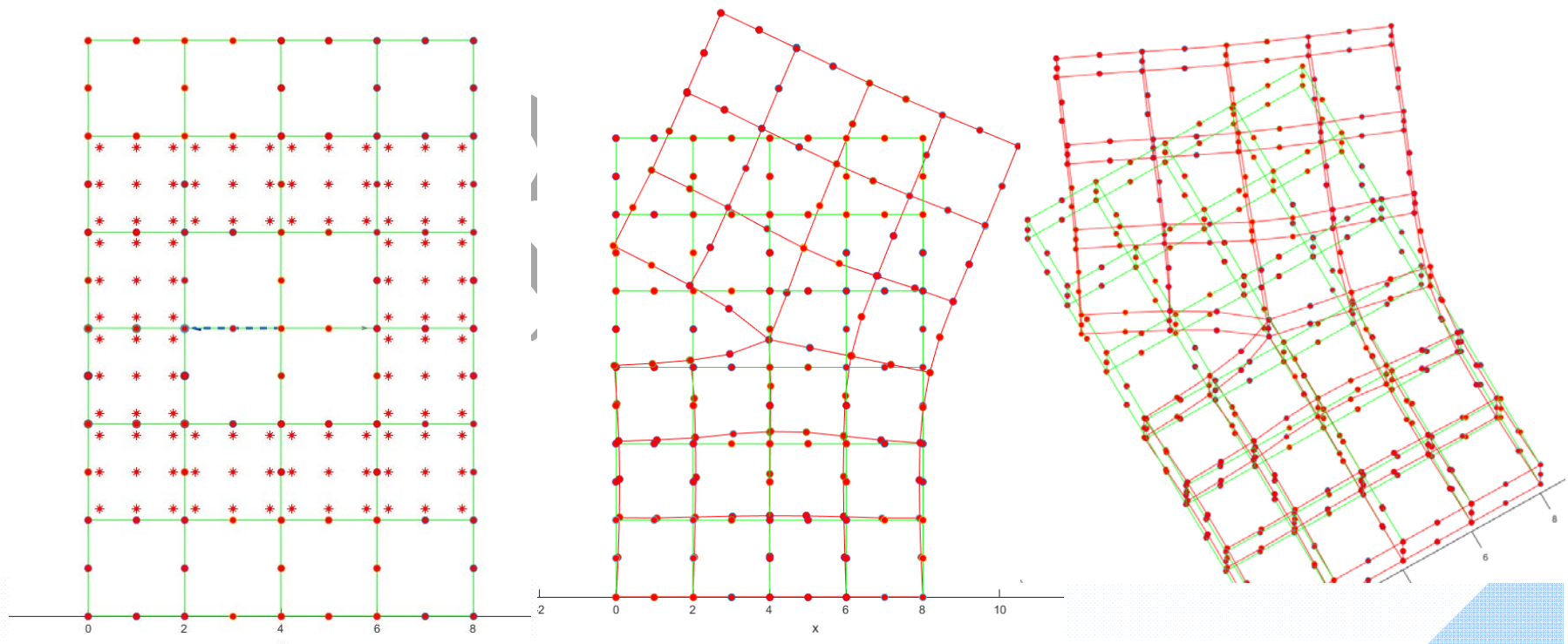


Stress Intensify Factor evaluation: Contour Integral(3D J-Integral)

- With the EDI method, a point-wise value of J-integral along a three-dimensional crack front can be calculated, and therefore the value of K along the crack front can be obtained.
- In order to do J-integral: Chiarelli-Frediani formulation for 20 node brick element

$$J_k = (J_L)_k + (J_A)_k \quad k = 1, 2, 3;$$

$$(J_A)_i(\Gamma) = \int_A \left\{ \frac{\partial \sigma_{xz}}{\partial z} \frac{\partial u}{\partial x} + \sigma_{xz} \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial \sigma_{yz}}{\partial z} \frac{\partial v}{\partial x} + \sigma_{yz} \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial \sigma_{zz}}{\partial z} \frac{\partial w}{\partial x} + \sigma_{zz} \frac{\partial^2 w}{\partial x \partial z} \right\} dA;$$





Stress Intensify Factor evaluation: Contour Integral(3D M- Integral)

- A more convenient way to determine the mixed-mode SIF values compared to the domain integral is the interaction integral which gives a more robust and actual results.
- The interaction integral in a 3D domain is a contribution of the domain integral in called actual field, an auxiliary field and an interacting field between the actual and auxiliary field.

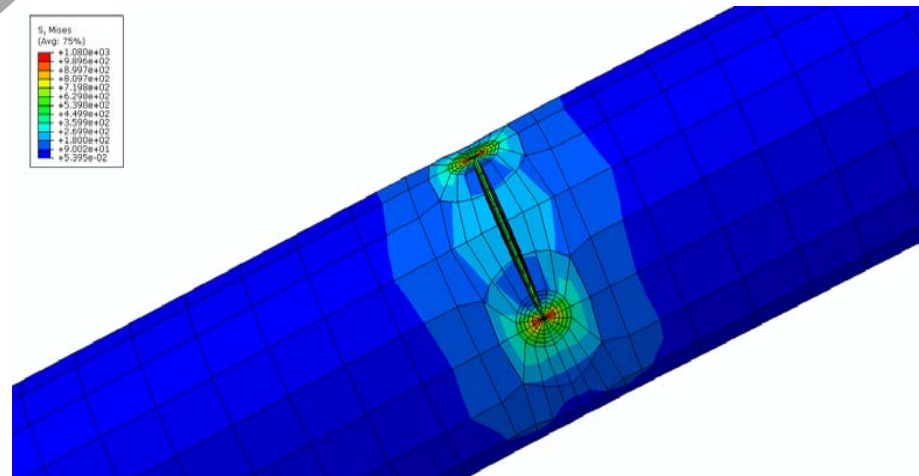
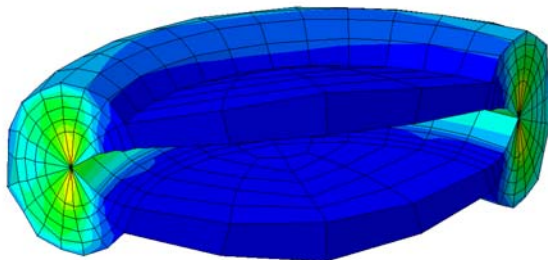
$$J^s = J(s) + J^{aux}(s) + M(s)$$

$$M(s) = \int_A \left[\sigma_{ij} \frac{\partial u_j^{aux}}{\partial x_1} + \sigma_{ij}^{aux} \frac{\partial u_j}{\partial x_1} - \sigma_{jk} \varepsilon_{jk}^{aux} \sigma_{1i} \right] \frac{\partial q}{\partial x_i} dV$$

- $J(s)$: actual field from the domain integral
- $J^{aux}(s)$: auxiliary field in the vicinity of a crack, containing the auxiliary stress, strain and displacement.
- $M(s)$: Interaction (M-Integral), that interacting the auxiliary and actual field are given by, without the components of traction.
- In ABAQUS using M-Integral with collapsed elements create very good results with optimum number of meshes.

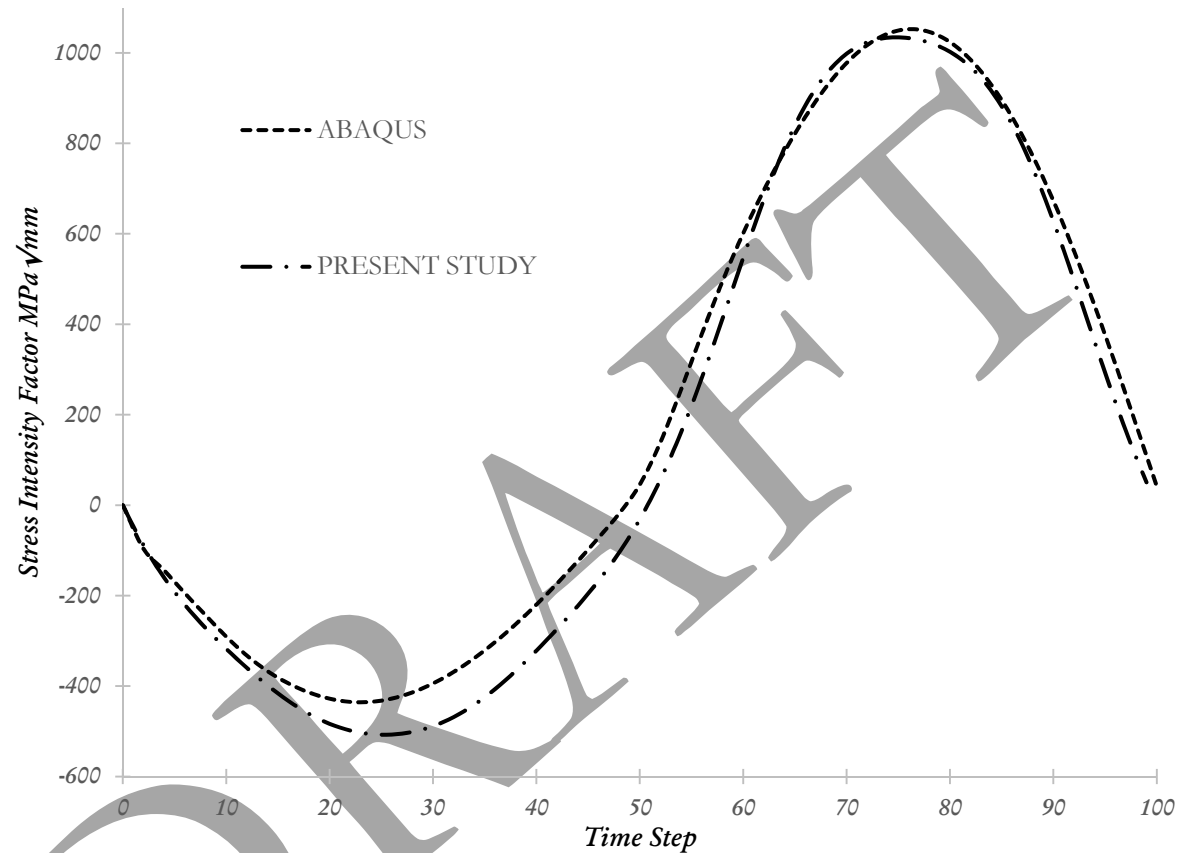


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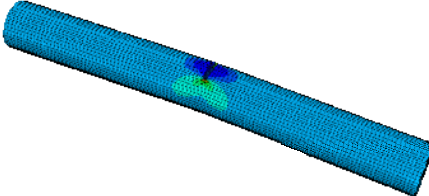
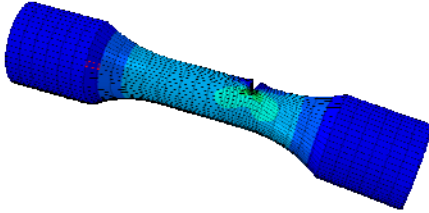
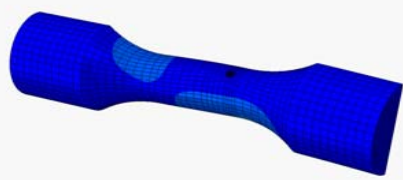
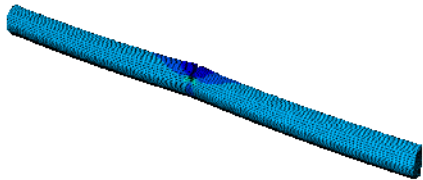
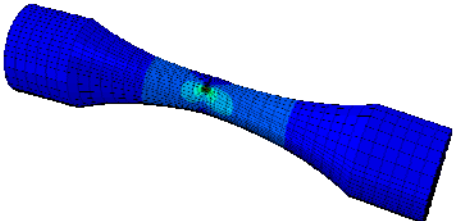
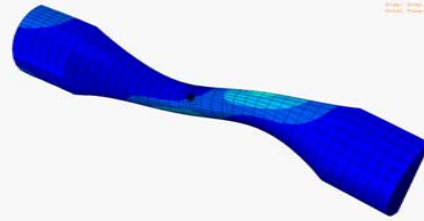
Verification of HBM with ABAQUS: Benchmark



SOLVER	ABAQUS[IMPLICIT]	HBM+M-INTEGRAL
	WALLCLOCK TIME (SEC) =3133 sec	156[HBM]+1374[M-INTEGRAL] =1530 sec



Results: static vs. dynamic

	<i>specimen core (bar) static simulation</i>	<i>specimen static simulation</i>	<i>specimen dynamic simulation (20 kHz)</i>
<i>Gaussian</i>			
<i>Hourglass</i>			

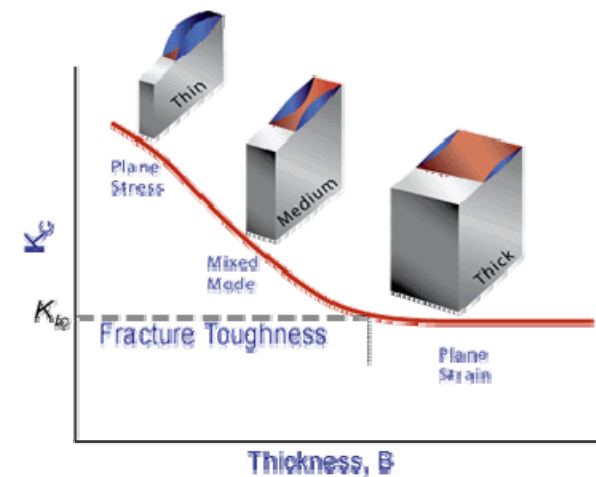
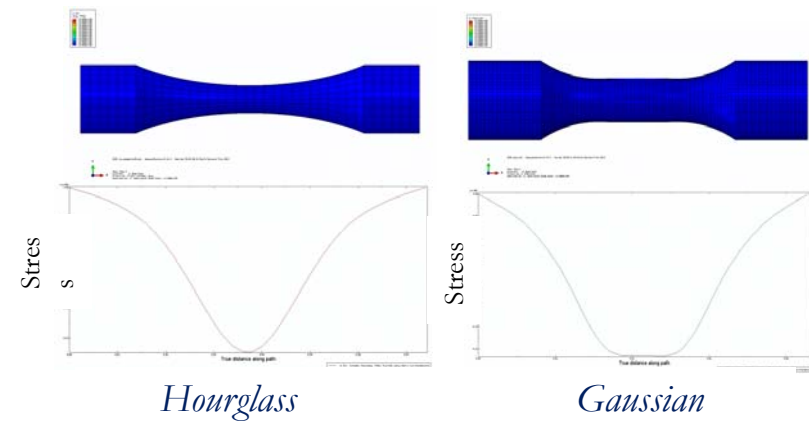
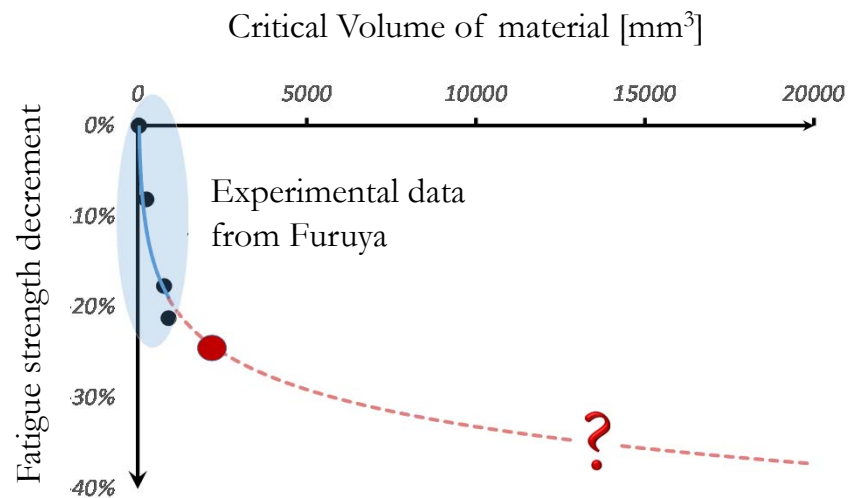
<i>specimen type</i>	<i>analytic static solutions</i>		<i>FEM solutions</i>			<i>error (percent)</i>	
	<i>Forman</i>	<i>Shih & Cai</i>	<i>bar static</i>	<i>specimen static</i>	<i>specimen dynamic</i>	<i>bar/specimen static</i>	<i>static/dynamic</i>
<i>Gaussian</i>	48.3	49.0	46.7	46.5	39.5	4.6	17.5
<i>Hourglass</i>	74.4	71.5	72.4	76.3	61.6	4.3	23.9

SIF values [MPa \sqrt{m}]



Discussion: size effects

Size effect in VHCF \rightarrow significant decrement of fatigue properties [Furuya, Murakami, Beretta].





Conclusions and Further research

Conclusions

- **An innovative procedure** for the effective and efficient computation of relevant SIFs in a cracked body in resonance condition was proposed. The procedure is based on the use of Reduced Order Models(CB-CMS), Harmonic Balance Method and Virtual Crack Closure Technique(VCCT),J-Integral and M-Integral.
- **The applicability and the potentialities** of the proposed hybrid method were successfully shown with a numerical example. It was found that a reduction factor of about 600 in terms of computational time can be achieved with the proposed procedure with no loss of accuracy.
- **Gaussian vs. Hourglass specimens:** significant different SIFs
- **SIF smaller in Gaussian specimen:** larger size induces a transition to plane strain condition
- **SIF evaluated with Gaussian specimens closer to critical SIF (material fracture toughness)**

Further research

- **Computation of threshold SIF** at the boundaries of the Optically-Dark-Area.



Publications and Courses

■ *Publications:*

- *Experimental-Numerical Assessment of Critical SIF from VHCF Tests*, Key Engineering Materials, Advances in Fracture and Damage Mechanics XV, Volume 713, pp.62-65, September 2016.
- *Numerical computation of stress intensity factor in ultrasonic Very-High Cycle Fatigue tests*, Key Engineering Materials -754 September 2017

■ *Courses:*

- Analysis of structures subjected to impulsive loading-4 credits [20 hours], Politecnico di Torino
- The Boundary Element Method for Anisotropic Bodies and Multilayered - 3 credits [20 hours], Politecnico di Torino
- Nonlinear structural dynamics-2credits [10 hours], Politecnico di Torino
- Managing Ph.D. Thesis as a Project - 2 credits [16 hours], Politecnico di Torino
- Applied probability and stochastic processes - 6 credits [30 hours], Politecnico di Torino
- Tools and applications of systems engineering - 6 credits [30 hours], Politecnico di Torino
- Topics In Internet & Society Interdisciplinary Studies - 4 credits [20 hours], Politecnico di Torino
- Programming in LabVIEW: Part 1 and Part 2- 8 credits [40 hours], Politecnico di Torino
- The redefinition of the International System of Units (SI)- 3 credits [15 hours], Politecnico di Torino
- Competitive funds for research: from the idea to the writing of the project- 2credits [10 hours], Politecnico di Torino
- Models and methods for the dynamics of mechanical components with contact interfaces-3 credits -[10 hours], Politecnico di Torino
- Probe scanning microscopy for physics and engineering-3 credits-[30 hours]-Politecnico di Torino
- TOTAL: 53 credits collected